Modern Physics

Final Exam

Name: _____

Student I.D.: _____

Short Answer (10 pts each)

Answer all problems in as thorough detail as possible. Be sure to include all your work. Partial credit will be given even if the answer is not fully correct.

1. Calculate the density of an object of one solar mass whose radius is the critical Schwarzschild radius. Compare this density with the nuclear density of approximately $2.3 \times 10^{17} kg/m^3$.

2. The Eiffel Tower in Paris is 300m tall. What is the fractional gravitational red shift due to this elevation difference?

3. Determine the charge in the rest mass of a system consisting of a proton and an electron as the two particles combine to form a hydrogen atom. The ionization energy of hydrogen is 13.6 eV.

4. Derive Stefan's law from Planck's law. Using that result, obtain an expression for the Stefan-Boltzmann constant σ in terms of known physical constants and determine its value.

A Partial List of Fundamental Constants

Constant	Symbol	Approximate Value
Speed of light in vacuum	c	3.00×10^{8}m/s
Permeability of vacuum	μ_0	12.6×10^{-7}H/m
Permittivity of vacuum	ϵ_0	8.85×10^{-12}F/m
Magnetic flux quantum	$\phi_0 = \frac{h}{2e}$	2.07×10^{-15}Wb
Electron mass	m_e	9.11×10^{-31}kg
Proton mass	m_p	1.673×10^{-27}kg
Neutron mass	m_n	1.675×10^{-27}kg
Proton-electron mass ratio	$\frac{m_p}{m_e}$	1836

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Answer Key for Exam A

Short Answer (10 pts each)

Answer all problems in as thorough detail as possible. Be sure to include all your work. Partial credit will be given even if the answer is not fully correct.

1. Calculate the density of an object of one solar mass whose radius is the critical Schwarzschild radius. Compare this density with the nuclear density of approximately $2.3 \times 10^{17} kg/m^3$.

   Answer: The mass of the sun is $2 \times 10^{30} kg$. The universal gravitational constant is $6.67 \times 10^{-11} N \cdot m^2/kg^2$.

   The Schwarzschild radius of one solar mass is

   $$\begin{aligned} R_c &= \frac{2(2 \times 10^{30} lg)(6.67 \times 10^{-11} N \cdot m62/kg^2)}{(3 \times 10^8 m/s)^2} \\ &= 2.96 \times 10^3 m \approx 3km \end{aligned}$$

   If the sun collapsed to a sphere of 3 km radius without loss of mass, it would then be a black hole. The mass density would be

   $$\rho = \frac{M}{V} = \frac{2 \times 10^{30} kg}{(\frac{4\pi}{3})(2.96 \times 10^3 m)^3} = 1.84 \times 10^{19} kg/m^3$$

2. The Eiffel Tower in Paris is 300m tall. What is the fractional gravitational red shift due to this elevation difference?

   Answer:

   $$\frac{\delta\nu}{\nu} = \frac{gH}{c^2} = \frac{(9.8 m/s^2)(300m)}{(3 \times 10^8 m/s)^2} = 3.27 \times 10^{-14}$$

3. Determine the charge in the rest mass of a system consisting of a proton and an electron as the two particles combine to form a hydrogen atom. The ionization energy of hydrogen is 13.6 eV.

   Answer: The rest mass of the proton is 1.672649×10^{-27}kg; that of the electron is 9.109534×10^{-31}kg. When the electron and proton combine to form a hydrogen atom, the ionization energy of 13.6 eV is released as ultraviolet radiation. The rest mass of the hydrogen atom is therefore smaller than the sum of the electron and proton rest masses by the amount

   $$\Delta M = \frac{(13.6\text{eV})(1.6 \times 10^{-19} J/\text{eV})}{(3 \times 10^8 m/s)^2} = 2.42 \times 10^{-35} kg$$

4. Derive Stefan's law from Planck's law. Using that result, obtain an expression for the Stefan-Boltzmann constant σ in terms of known physical constants and determine its value.

   Answer: The radiance

A Partial List of Fundamental Constants

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Answer Key for Exam B

Short Answer (10 pts each)

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1. Calculate the density of an object of one solar mass whose radius is the critical Schwarzschild radius. Compare this density with the nuclear density of approximately $2.3 \times 10^{17} kg/m^3$.

   Answer: The mass of the sun is $2 \times 10^{30} kg$. The universal gravitational constant is $6.67 \times 10^{-11} N \cdot m^2/kg^2$.

   The Schwarzschild radius of one solar mass is

   $$\begin{aligned} R_c &= \frac{2(2 \times 10^{30} lg)(6.67 \times 10^{-11} N \cdot m62/kg^2)}{(3 \times 10^8 m/s)^2} \\ &= 2.96 \times 10^3 m \approx 3km \end{aligned}$$

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3. Derive Stefan's law from Planck's law. Using that result, obtain an expression for the Stefan-Boltzmann constant σ in terms of known physical constants and determine its value.

   Answer: The radiance

4. The Eiffel Tower in Paris is 300m tall. What is the fractional gravitational red shift due to this elevation difference?

   Answer:

   $$\frac{\delta\nu}{\nu} = \frac{gH}{c^2} = \frac{(9.8 m/s^2)(300m)}{(3 \times 10^8 m/s)^2} = 3.27 \times 10^{-14}$$

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Answer Key for Exam C

Short Answer (10 pts each)

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   $$\frac{\delta\nu}{\nu} = \frac{gH}{c^2} = \frac{(9.8m/s^2)(300m)}{(3\times 10^8 m/s)^2} = 3.27\times 10^{-14}$$

2. Calculate the density of an object of one solar mass whose radius is the critical Schwarzschild radius. Compare this density with the nuclear density of approximately $2.3\times 10^{17} kg/m^3$.

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   Answer: The radiance

4. Determine the charge in the rest mass of a system consisting of a proton and an electron as the two particles combine to form a hydrogen atom. The ionization energy of hydrogen is 13.6 eV.

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Answer Key for Exam D

Short Answer (10 pts each)

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